Inductive Inferences in *CL* Diagrams

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CL diagrams – the abbreviation of *C*ubus *L*ogicus – are inspired by J.C. Lange's logic machine from 1714 [2]. In recent times, Lange's diagrams have been used for extended syllogistics, bitstring semantics, analogical reasoning and many more [3]. Recently it has been proved that *CL* diagrams can also form a logical system that is sound and complete [1].

A typical *CL* diagram consists of structural elements and content elements. Structural elements are solid boxes nested like a binary tree and dotted boxes as auxiliary lines that indicate the number of boxes still subsumed. The content elements such as arrows depict logical operations over the solid boxes. The solid boxes in Fig. 1 show 3 classes, T, H, NH, and 4 individuals, X, Y, Z, W. Detailed definitions can be found in [1].

Here we will demonstrate inductive reasoning in *CL* with a simple example of an inductive syllogism:

75% of all humans are taller than 2 feet.
Gareth is a human.

Therefore, Gareth is taller than 2 feet. ∴

In Fig. 1, 'T' represents set of the heights of all the living being, 'H' the set of humans, 'NH' non-humans. The solid boxes in Fig. 1 can be divided into smaller boxes using blue dotted lines (see Fig. 2). Each blue box in Fig. 2 represents 25% of the entire set. For the convention purpose, here we take any solid box

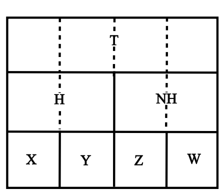

Fig. 1.

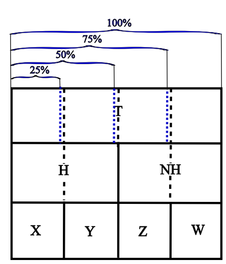

Fig. 2.

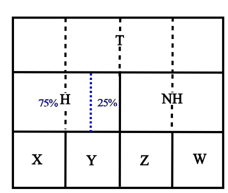

Fig. 3.

as the complete 100% and if it is divided in 4 parts then each of the part is 25%. This is not a strict convention. We can divide the boxes in any way. The only thing that we have to keep in mind is that the addition of all the divided parts of a solid box is exactly 100%. For example, in Fig. 3, the solid box H is divided into two parts.

Now let us assume that the maximum height of the living being is 16ft. So, each blue box represent the intervals as shown in Fig. 4. These blue boxes can further be divided as shown in Fig. 5. Fig. 6 represents the boxes of all heights greater than 2 feet.

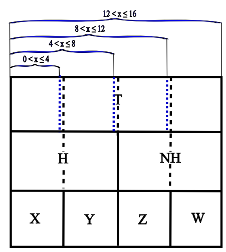

Fig. 4.

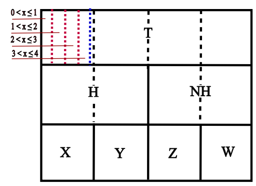

Fig. 5.

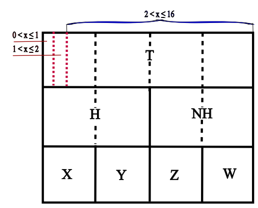

Fig. 6.

In inductive reasoning we use arrows and coloured boxes as content elements to represent propositions. The broken arrow in Fig. 7 goes from the 75% box in H (arrow shaft) to the 2 ⩽ 16 box in T (arrowhead), indicating that 75% of all humans are taller than 2 feet. The proposition "Gareth is a human" is represented by including 'Gareth' directly under the box for humans, e.g. Fig. 8. Finally, we use the following inference rule to obtain Fig. 9 which represent that Gareth (arrow shaft) is taller than 2 feet (arrowhead).

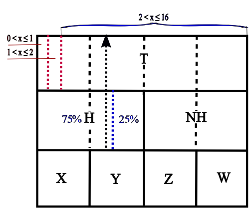

Fig. 7.

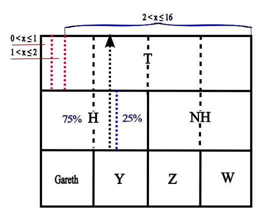

Fig. 8.

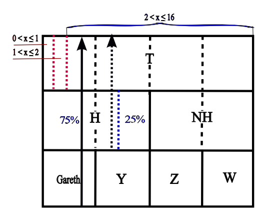

Fig. 9.

Inference Rule: If a broken arrow connects the maximum collection of smallest subboxes of a box, say A, to the maximum collection of smallest subboxes of another box B then a solid arrow can be drawn from the individual box that is included in A to the maximum collection of smallest subboxes of B.

'Broken arrow' is a representation of 'observation', whereas a 'solid arrow' is a representation of the 'conclusion' that has been infered from our observation. In inductive reasoning we arrive at some conclusion through observation. This is similar in our example: Like in the 'inference rule', we first observe the position of the broken arrow then we arrive at our conclusion that Gareth's height is more than 2 feet.

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